Comments on "A simple method to compute economic order quantities"

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Abstract: This study examines Teng [1] that was published in European Journal of Operational Research 198, 351-353 to provide a simpler solution procedure for EOQ/EPQ inventory models by using Arithmetic-Geometric-Mean inequality. We will point out Teng [1] adopted the fill rate from Wee, Wang and Chung [2], European Journal of Operational Research 194, 336-338 to convert two-variable minimum problems into one-variable problems. Hence, his simpler solution procedure is standing on the shoulder of a giant. There are three related papers: Cárdenas-Barrón [3, 4] and Leung [5] to further discussed Teng [1]. We will provide our comments for these four related papers.

Keywords: inventory models, arithmetic-geometric-mean (AGM) inequality, without calculus, fill rate

1. Introduction

In the revolution of academic history, if a research can provide a simple solution process to replace previously established lengthy and hard-working procedure, then his breaking through achievement will have great impact on scholastic society. Teng [1] announced that he created a new solution approach that is superior to Chang et al. [6], Grubbström and Erdem [7], Ronald et al. [8], Sphicas [9], Wee and Chung [10], Wu and Ouyang [11], and others. This astonished improvement arouses our attention which deserves a careful study.

Until now, there are 46 papers that had been referred Teng [1] in their References. We divide them into the following three categories.

(A) There are three comprehensive literature survey papers: Andriolo et al. [12], Drake and Marley [13] and Shekarian et al. [14].

(B) Three papers are really related to Teng [1]: Cárdenas-Barrón [3, 4] and Leung [5]

(C) The rest 40 papers only mentioned Teng [1] in their Introduction and then they concentrated on their new inventory models. We list them in the following: Teng and Goyal [15], Chang and Ho [16], Dellino et al. [17], Widyadana and Wee [18], Cárdenas-Barrón [19], Cárdenas-Barrón et al. [20], Chang and Ho [21], Pasandideh et al. [22], Teng et al. [23], Widyadana et al. [24], Chang [25], Chung [26], Li et al. [27], Yadav et al. [28, 29], Chung [30, 31], Gambini et al. [32], Lou and Wang [33], Teng et al. [34], Wee et al. [35], Yadav et al. [36], Chang [37], Chen et al. [38], Chern et al. [39], Gambini et al. [40], Liao et al. [41], Teng et al. [42], Nagoor Gani and Raja Dharik [43], Teng and Hsu [44], Chen et al. [45], Kumar and Aarya [46], Niknamfar and Niaki [47], Gong et al. [48], Goyal et al. [49], Kojić [50], Liao et al. [51], Rajan and Uthayakumar [52] and Vidal-Carreras et al. [53].

2. Notation and assumptions

To be compatible with Teng [1], we use the same notation and assumptions as his paper.

Notation

A the ordering cost per replenishment
h the holding cost per unit and per unit of time
d the constant demand per unit of time
p the production rate per unit of time
Q the order quantity per replenishment
Q^* the optimal order quantity
r the fill rate, that is the ratio between the inventory period and one replenishment cycle
T the planning horizon for one replenishment
v the backorder cost per unit per unit of time

Assumptions
(1) The lead time is zero so that replenishment is instantaneous.
(2) There is no quantity discount.
(3) Both the initial and the ending inventory levels are zero so that there is no salvage value.
(4) $TC_1(Q)$ is the EOQ model without backorders.
(5) $TC_2(Q)$ is the EOQ model with backorders.
(6) $TC_3(Q)$ is the EPQ model with backorders.

3. Review of Teng [1]

Teng [1] mentioned that he applied the Arithmetic-Geometric-Mean (AGM) inequality, which is

$$\frac{a+b}{2} \geq \sqrt{ab}, \tag{1}$$

for any two positive real numbers. The equality holds if and only if $a = b$.
Teng [1] considered three inventory models: (a) EOQ model without backorders, (b) EOQ model with backorders, and (c) EPQ model with backorders.
For EOQ model without backorders, the objective function,

$$TC_1(Q) = \frac{Ad}{Q} + \frac{hQ}{2}, \tag{2}$$

Teng [1] applied AGM to derive

$$TC_1^*(Q) = \sqrt{2Adh}, \tag{3}$$

and $Q^* = \sqrt{2Ad/h}$.

For EOQ model with backorders, Teng [1] cited the objective function from Wee et al. [2],

$$TC_2(Q) = \frac{Ad}{Q} + \frac{Q}{2}\left[hr^2 + v(1-r)^2\right], \tag{4}$$

Teng [1] applied AGM to derive

$$TC_2^*(Q) = \sqrt{2Ad\left[hr^2 + v(1-r)^2\right]}, \tag{5}$$

and $Q^* = \sqrt{2Ad/\left[hr^2 + v(1-r)^2\right]}$.

For EPQ model with backorders, Teng [1] cited the objective function from Wee et al. [2],

$$TC_3(Q)=\frac{Ad}{Q}+\frac{Q}{2}\left(\frac{p-d}{p}\right)\left[hr^2+v(1-r)^2\right], \tag{6}$$

Teng [1] applied AGM to derive

$$TC_3^*(Q)=\sqrt{2Ad(p-d)\left[hr^2+v(1-r)^2\right]/p}, \tag{7}$$

and $Q^*=\sqrt{2Ad/\left\{(p-d)\left[hr^2+v(1-r)^2\right]\right\}}$.

Teng [1] mentioned that his solution approach is easy-to-apply and simple-to-understand. We quote his assertion in his Conclusions "By using the proposed method, we also can obtain the global minimum solutions much easier and simpler than the method of computing perfect square established by Chang et al. [6], Grubbström and Erdem [7], Ronald et al. [8], Sphicas [9], Wee and Chung [10], Wu and Ouyang [11], and others."

4. Our comments for Teng [1]

For EOQ model with backorders, the objective function of Wee et al. [2],

$$TC_2(Q,r)=\frac{Ad}{Q}+\frac{Q}{2}\left[hr^2+v(1-r)^2\right], \tag{8}$$

and then Wee et al. [2] applied Cost-Difference Comparison Method (CDCM) to derive

$$r^*=\frac{v}{h+v}. \tag{9}$$

In Teng [1], he never mentioned the result of Equation (9).

Grubbström and Erdem [7], and Ronald et al. [8] considered EOQ model with backorders, in different expression,

$$C(Q,B)=\frac{D}{B+Q}\left(\frac{b}{2D}B^2+\frac{h}{2D}Q^2+K\right). \tag{10}$$

We transfer their expressions to the same notation as Wee et al. [2], then

$$C(rQ,(1-r)Q)=\frac{Ad}{Q}+\frac{h}{2}(rQ)^2+\frac{v}{2}[(1-r)Q]^2. \tag{11}$$

Grubbström and Erdem [7], and Ronald et al. [8] were facing two-variable minimum problem. Teng [1] adopted result from Wee et al. [2] to convert his two-variable problem into one-variable problem, and then Teng [1] asserted that his solution approach is simpler than Grubbström and Erdem [7], and Ronald et al. [8]. However, Teng [1] did not clearly inform readers that he used Equation (9) during his conversion. Hence, his simpler announcement is a bias assertion.

For EPQ model with backorders, Chang et al. [6] considered the following minimum problem, in different expression,

$$C(Q,B)=\frac{D}{B+Q}\left(\frac{b}{2D}B^2+\frac{h}{2D}Q^2+K\rho\right)+cD, \tag{12}$$

We transfer their expressions to the same notation as Wee et al. [2], then

$$C(rQ,(1-r)Q)=\frac{Ad}{Q}+\frac{h}{2}(rQ)^2\left(\frac{p-d}{p}\right)+\frac{v}{2}[(1-r)Q]^2\frac{p-d}{p}+cd, \tag{13}$$

such that Chang et al. [6] was facing two-variable minimum problem.

In Wee et al. [2], they did not apply CDCM to find r^* for EPQ model with backorders. Instead, Wee et al. [2] claimed that by the similar argument for EOQ model with backorders, they can derive $r^* = v/(h+v)$.

Teng [1] quoted the results from Wee et al. [2] to simplify $TC_3(Q,r)$ with two variables Q and r to $TC_3(Q)$ with one variable, Q.

The superiority of Teng [1] with $TC_3(Q)$ to Chang et al. [6] with $C(rQ,(1-r)Q)$ is a misleading assertion, because Teng [1] implicitly applied $r^* = v/(h+v)$ during his reduction.

5. Review of Cárdenas-Barrón [3]

Cárdenas-Barrón [3] extended AGM mentioned by Teng [1] to a more general setting, for $a_1, a_2, \ldots, a_n$ being n positive real numbers,

$$\frac{\sum_{k=1}^{n} a_k}{n} \geq \left(\prod_{k=1}^{n} a_k\right)^{\frac{1}{n}} \tag{14}$$

with equality if and only if $a_1 = a_2 = \ldots = a_n$. Cárdenas-Barrón [3] also recalled the Cauchy-Bunyakovsky-Schwarz (CBS) inequality,

$$\sum_{k=1}^{n} a_k^2 \sum_{k=1}^{n} b_k^2 \geq \left(\sum_{k=1}^{n} a_k b_k\right)^2 \tag{15}$$

For two sets of real numbers $a_1, a_2, \ldots, a_n$ and $b_1, b_2, \ldots, b_n$, with equality if two sets of real numbers are proportional as $(a_1/b_1) = (a_2/b_2) = \ldots = (a_n/b_n)$.

Cárdenas-Barrón [3] considered the same EOQ model with backorders, but in different expression, where Q is the order quantity and B is the maximum backorders level such that the maximum inventory level is $Q-B$,

$$TC(Q,B) = \frac{Ad}{Q} + \frac{h(Q-B)^2}{2Q} + \frac{vB^2}{2Q}, \tag{16}$$

Cárdenas-Barrón [3] rewrote Equation (16) as

$$TC(Q,B) = \frac{Ad}{Q} + \frac{Q}{2}\left\{\left[\sqrt{h}\left(1-\frac{B}{Q}\right)\right]^2 + \left[\sqrt{v}\,\frac{B}{Q}\right]^2\right\}. \tag{17}$$

Cárdenas-Barrón [3] adopted a genius formula

$$1 = \left\{\left(\sqrt{\frac{v}{h+v}}\right)^2 + \left(\sqrt{\frac{h}{h+v}}\right)^2\right\}, \tag{18}$$

to further rewrite Equation (18) as

$$TC(Q,B) = \frac{Ad}{Q} + \frac{Q}{2}\left\{\left[\sqrt{h}\left(1-\frac{B}{Q}\right)\right]^2 + \left[\sqrt{v}\,\frac{B}{Q}\right]^2\right\}\left\{\left(\sqrt{\frac{v}{h+v}}\right)^2 + \left(\sqrt{\frac{h}{h+v}}\right)^2\right\}. \tag{19}$$

Cárdenas-Barrón [3] applied CBS inequality to Equation (19) to derive

$$TC(Q,B) \geq \frac{Ad}{Q} + \frac{Q}{2}\left\{\left[\sqrt{h}\left(1-\frac{B}{Q}\right)\right]\sqrt{\frac{v}{h+v}} + \left[\sqrt{v}\frac{B}{Q}\right]\sqrt{\frac{h}{h+v}}\right\}^2, \quad (20)$$

and the equality holds when $h(1-(B/Q)) = v(B/Q)$ which is identical to the fill rate $(Q-B)/Q = v/(h+v)$ proposed by Wee et al. [2].

Cárdenas-Barrón [3] simplified Equation (20) to obtain

$$TC(Q, B = hQ/(h+v)) = \frac{Ad}{Q} + \frac{Q}{2}\sqrt{\frac{hv}{h+v}}. \quad (21)$$

Based on Equation (21), Cárdenas-Barrón [3] derived

$$Q^* = \sqrt{\frac{2Ad(h+v)}{hv}}, \quad (22)$$

and

$$TC(Q^*, B^* = hQ^*/(h+v)) = \sqrt{\frac{2Adhv}{h+v}}. \quad (23)$$

We can claim that Cárdenas-Barrón [3] provided a patchwork for Teng [1] for applying CBS inequality to solve EOQ models with backorders.

Moreover, for EPQ model with backorders,

$$TC(Q,B) = \frac{Ad}{Q} + \frac{h(Q\delta - B)^2}{2Q\delta} + \frac{vB^2}{2Q\delta}, \quad (24)$$

with $\delta = 1-(d/p)$, Cárdenas-Barrón [3] repeated his solution approach for EOQ models to solve his EPQ models. Cárdenas-Barrón [3] overlooked Sphicas [9] already used $\hat{A} = A\delta$ and $\hat{Q} = Q\delta$ to rewrite Equation (24) as

$$TC(\hat{Q},B) = \frac{\hat{A}d}{\hat{Q}} + \frac{h(\hat{Q}-B)^2}{2\hat{Q}} + \frac{vB^2}{2\hat{Q}}. \quad (25)$$

If we neglect “cap” of Equation (25), then Equation (25) is identical to Equation (16) such that findings from EOQ model can be directly applied for EPQ model, without the lengthy repeated computation proposed by Cárdenas-Barrón [3].

6. Review of Cárdenas-Barrón [4]

Cárdenas-Barrón [4] mentioned that

$$\frac{Z_1 + Z_2 + \ldots + Z_n}{n} \geq \sqrt[n]{Z_1 Z_2 \ldots Z_n} \quad (26)$$

where $Z_1, Z_2, \ldots, Z_n$ are functions of variables $x_1, x_2, \ldots, x_2$, then the following three restrictions:

(i) All functions $Z_1, Z_2, \ldots, Z_n$ are positive functions,

(ii) The multiplication of all functions is a constant, that is, $Z_1 Z_2 \ldots Z_n$ is independent of $x_1, x_2, \ldots, x_2$,

(iii) When these functions are assumed to be equal, that is, $Z_1 = Z_2 = \ldots = Z_n$, then the substituted system can be solved, that must be hold, before applying AGM to solve minimum problems as applied by Teng [1], and then he criticized Wee et al. [2] did not find the backorders level.

Cárdenas-Barrón [4] referred the fill rate $r = v/(h+v)$ derived by Wee et al. [2] by CDCM to claim

$$B = Q(1-r) = \left(\frac{h}{h+v}\right)Q, \tag{27}$$

$$Q = \sqrt{\frac{2Ad(h+v)}{hv}}, \tag{28}$$

and

$$TC = \sqrt{\frac{2Adhv}{h+v}}. \tag{29}$$

However, in Cárdenas-Barrón [4], he did not inform us how to derive Equations (28) and (29). Similar problems occur for EPQ model with backorders of Cárdenas-Barrón [4].

We find that Wee et al. [2] did not write the backorder level, B for their EOQ and EPQ models. However, Wee et al. [2] already found Q^* and r^* such that the derivation of the maximum backorder level as

$$B^* = Q^*(1-r^*), \tag{30}$$

becomes a trivial exercise.

The similar problem occurs for his criticism for the backorder level of Wee et al. (2009) for EPQ model with backorders. Cárdenas-Barrón [4] provided a patch work for the missing discussion for fill rate in Wee et al. [2] for EPQ models and then Cárdenas-Barrón [4] still obtained the fill rate $r = v/(h+v)$. However, the optimal order quantity for EPQ models

$$Q = \sqrt{\frac{2Ad(h+v)}{(1-(d/p))hv}}, \tag{31}$$

still suddenly appeared in Cárdenas-Barrón [4] without any explanation.

7. Review of Leung [5]

There are two comments for Leung [5] with respect to Teng [1]. For the first comment, Leung [5] noticed that in Teng [1], the fill rate, r, is not treated. Instead to cite from Wee et al. [2] for $r = v/(h+v)$, Leung [5] considered to minimize the last term of Equation (4) used by Teng [1], to denote it as a new expression, say $\eta(r)$, with

$$\eta(r) = hr^2 + v(1-r)^2, \tag{32}$$

to derive

$$\eta(r) = (h+v)\left(r - \frac{v}{h+v}\right)^2 + \frac{hv}{h+v}. \tag{33}$$

Hence, Leung [5] obtained the optimal fill rate, $r^* = v/(h+v)$.

For the second comment, Leung [5] mentioned that for EOQ model with backorders, then the objective function should contained two variables: Q and r such that Leung [5] changed Equation (4) to the following

$$TC(Q,r) = \frac{Ad}{Q} + \frac{Q}{2}\left[hr^2 + v(1-r)^2\right]. \tag{34}$$

Leung [5] pointed out that the solution procedure should be divided into three parts: Situation (a): $0 < r < 1$, Situation (b): $r = 0$, and Situation (c): $r = 1$.

Under three different situations, Leung [5] found

$$TC^*_{(a)} \equiv TC\left(Q^*, r^*\right) = \sqrt{\frac{2Adhv}{h+v}}, \tag{35}$$

$$TC^*_{(b)} \equiv TC\left(Q^*_{(b)}, r = 0\right) = \sqrt{2Adv}, \tag{36}$$

and

$$TC^*_{(a)} \equiv TC\left(Q^*_{(c)}, r = 1\right) = \sqrt{2Adh}. \tag{37}$$

Leung [5] compared findings of Equations (35-37) to imply that $TC^*_{(a)} < TC^*_{(b)}$ and $TC^*_{(a)} < TC^*_{(c)}$ to conclude that $TC^*_{(a)}$ is the global minimum.

We will show that the second comment of Leung [5] is unnecessary. We recall the first comment of Leung [5] by algebraic method to derive $r^* = v/(h+v)$ for $0 \le r \le 1$ such that there is unnecessary to divide $0 \le r \le 1$ into three situations as Situation (a): $0 < r < 1$, Situation (b): $r = 0$, and Situation (c): $r = 1$.

We can offer a reasonable explanation why did Leung [5] assume three different situations: Situation (a): $0 < r < 1$, Situation (b): $r = 0$, and Situation (c): $r = 1$.

If researchers used calculus to find $r^* = v/(h+v)$, because calculus only handles interior points that is $0 < r < 1$ and then researchers will have three different objective functions: one for interior points, and two for two boundaries, $r = 0$ and $r = 1$. For these three objective functions, each one have its own local minimums and then researchers have to compare three local minimums to decide which one is the global minimum.

However, Leung [5] applied algebraic method without calculus such that there is only one objective function that is suitable for $0 \le r \le 1$ to consider three different situations is tedious and useless.

8. Conclusion

Teng [1] adopted findings of Wee et al. [2] to change two-variable minimum problems into one-variable minimum problems, and then Teng [1] claimed that his solution approach is simpler than several papers dealt with two-variable setting. Teng [1] overlooked that he already stands on the shoulder of a giant such that his comparison of his solution approach with previous paper is unfair and misleading.

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